(4 pnts) **Name/Surname:**

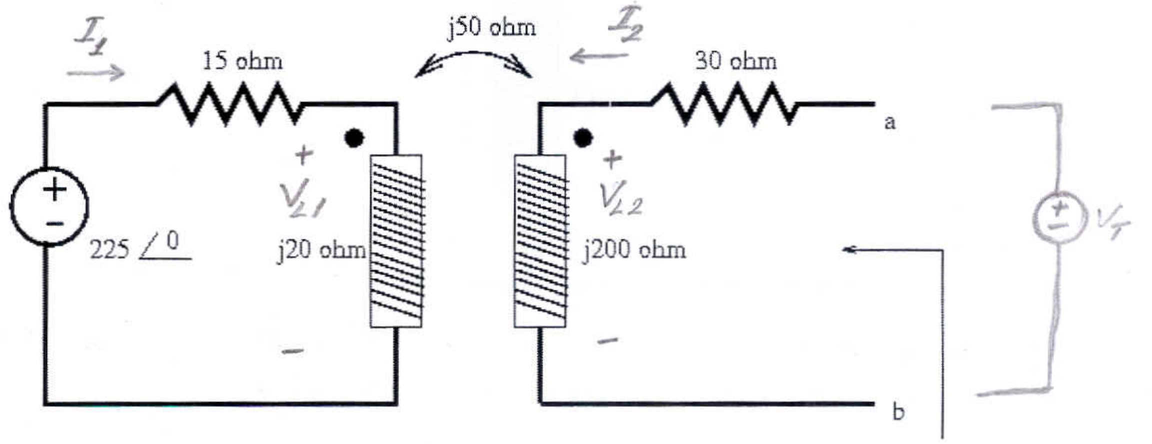
ID:

EE 210 – Introduction to Electrical Engineering – Midterm 2
May 12, 2010

Duration: 120 min, Closed book and notes. Calculators are allowed.

Illegible answers will not be graded.

1. (24pnts) Find the Thevenin equivalent circuit in phasor domain for terminals a-b.



open circuit:

$I_2 = 0$

$V_{22} = V_{Th} = j50\, I_1$

$225 = 15 I_1 + j20 I_1 + \cancel{j50 I_2} \to 0$

$I_1 = \dfrac{225}{15+j20}$

$V_{Th} = j50 \dfrac{225}{15+j20} \cdot \dfrac{15-j20}{15-j20} = \dfrac{225e3 + j168750}{625} = 360 + j270$

Remove the voltage source and apply V_T to terminals a-b:

$V_T = I_2(30+j200) + I_1\, j50$

$0 = I_1(15+j20) + I_2\, j50 \quad \times \dfrac{-j50}{15+j20}$

$+$

$V_T = I_2\underbrace{\left[(30+j200) + \dfrac{2500}{15+j20}\right]}$

$Z_{Th} = 30 + j200 + \dfrac{2500}{15+j20}\, \dfrac{15-j20}{15-j20}$

$= 30 + j200 + 60 - j80$

$= 90 + j120$

o.c. V_{Th} 12 $\longrightarrow$ 8 Just KVL's.

s.c. Z_{Th} or i_{sc} 12 $\longrightarrow$ 8

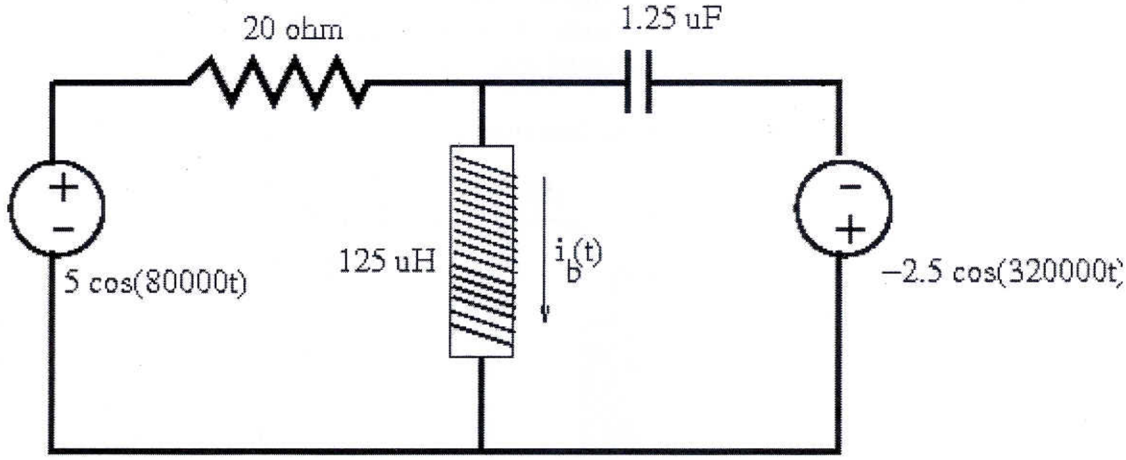
Computation error −4

$I_2 = 0$ in o.c. missed −6

$V_{Th} = 360 + j270$

$Z_{Th} = 90 + j120$

2. (24pnts) Find the steady state expression for $i_b(t)$.



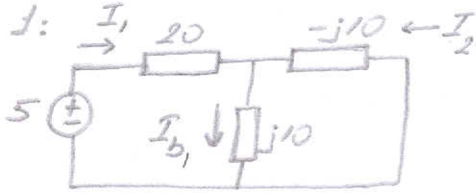
Since the 2 voltage sources operate at different frequencies, we need to use the superposition principle:

ω	$125\,\mu H$	$1.25\,\mu F$
80k	$j10$	$\frac{1}{j0.1} = -j10$
320k	$j40$	$\frac{1}{j0{,}4} = -j2{,}5$

Superposition principle in phasor domain 12pnts

Solution 6+6 pnts.

Source 1:



$5 = I_1(20+j10) + I_2 j10 \qquad I_{b_1} = I_1 + I_2$

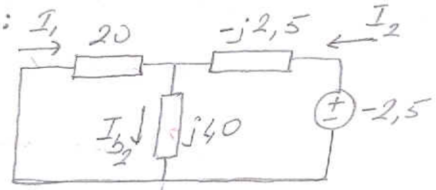
$0 = I_2(-j10+j10) + I_1 j10$

$\Rightarrow I_1 = 0 \Rightarrow I_2 = \frac{5}{j10} = -j0{,}5$

$\Rightarrow I_{b_1} = -j0{,}5 \Rightarrow i_{b_1}(t) = Re\{-j0{,}5\, e^{j80000t}\}$

$= 0{,}5 \cos(80000t - \frac{\pi}{2})$

Source 2:



$-2{,}5 = I_2 j37{,}5 + I_1 j40 \qquad I_{b_2} = I_1 + I_2$

$+\quad 0 = I_1 20 + I_2 j40 \quad \times -j2$

$-2{,}5 = I_2(80 + j37{,}5)$

$I_2 = \frac{-2{,}5}{80+j37{,}5} \Rightarrow I_1 = -I_2 j2 = \frac{j5}{80+j37{,}5}$

$I_{b_2} = \frac{-2{,}5+j5}{80+j37{,}5} \cdot \frac{80-j37{,}5}{80-j37{,}5} = -0{,}0016 + j0{,}0633$

$i_{b_2}(t) = Re\{(-0{,}0016 + j0{,}0633)\, e^{j320000t}\}$

$= Re\{0{,}0633\, e^{j0{,}508\pi}\, e^{j320000t}\}$

$= 0{,}0633 \cos(320000t + 0{,}508\pi)$

$0{,}508\pi = 1{,}5961$

$i_b(t) =$

$0{,}5 \cos(80000t - \frac{\pi}{2}) +$

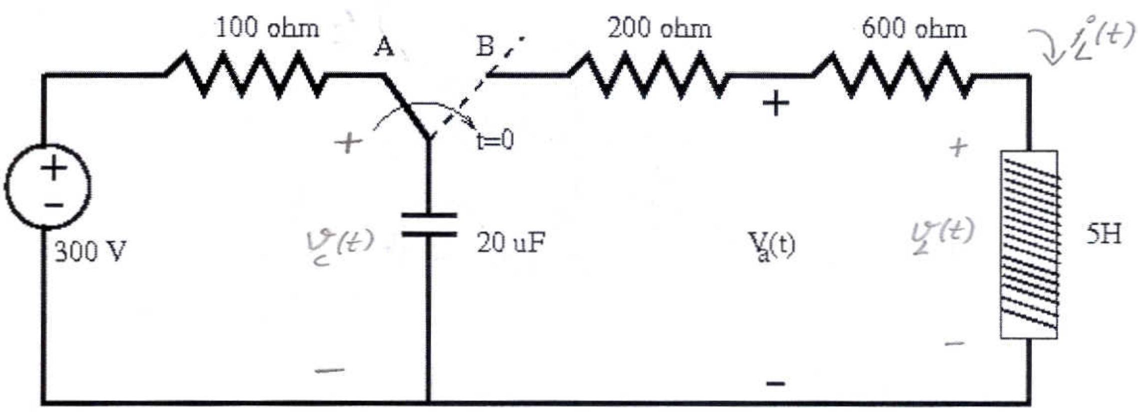
$0{,}0633 \cos(320000t + 0{,}508\pi)$

3. (6+6+12pnts) The switch in the circuit shown has been in position A for a long time.

a. $V_a(0^+) = ?$

b. $\left.\dfrac{dV_a(t)}{dt}\right|_{t=0^+} = ?$

c. $V_a(t) = ?$



Continuous circuit parameters

$-i_L = C\dfrac{dv_C}{dt}$; v_C

$v_L = L\dfrac{di_L}{dt}$; i_L

a) $v_C(0^-) = v_C(0^+) = 300$ V (Capacitor o.c. in switch position A)

6 $i_L(0^-) = i_L(0^+) = 0$ A $\Rightarrow$ $v_a(0^+) = v_C(0^+) = 300$ V

& $v_L(0^+) = 300$ V

b) $v_a = v_C - 200\, i_L$

6

$$\frac{dv_a}{dt} = \frac{dv_C}{dt} - 200\frac{di_L}{dt} = \frac{-i_L}{C} - 200\frac{v_L}{L} \Rightarrow \left.\frac{dv_a}{dt}\right|_{t=0^+} = \frac{-i_L(0^+)}{C} - 200\frac{v_L(0^+)}{L}$$

$$= 0 - 200\frac{300}{5} = -12000$$

c) KVL for switch position B:

$$v_C = 800\, i_L + 5\frac{di_L}{dt} \Rightarrow \frac{dv_C}{dt} = 800\frac{di_L}{dt} + 5\frac{d^2 i_L}{dt^2} \Rightarrow \frac{-i_L}{C} = 800\frac{di_L}{dt} + 5\frac{d^2 i_L}{dt^2} \Rightarrow$$

$$\Rightarrow \ddot{i}_L + \underbrace{160}_{2\alpha}\dot{i}_L + \underbrace{10000}_{\omega_0^2} i_L = 0 \qquad \alpha^2 - \omega_0^2 = -3600 < 0 \text{ underdamped.}$$

$\omega_d = 60$

soln form or ODE : 6

type of soln: 6 (damping)

numeric result: 5 (bonus)

$$i_L(t) = e^{-80t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$v_a(t) = 600\, i_L(t) + 5\, i_L'(t)$$

$$= 600\, e^{-80t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) + 5(-80)e^{-80t}(\quad) + 5e^{-80t}(-B_1\omega_d \sin\omega_d t + B_2\omega_d \cos\omega_d t)$$

$$= 200\, e^{-80t}(\quad) + 5e^{-80t}(-B_1\omega_d \sin\omega_d t + B_2\omega_d \cos\omega_d t)$$

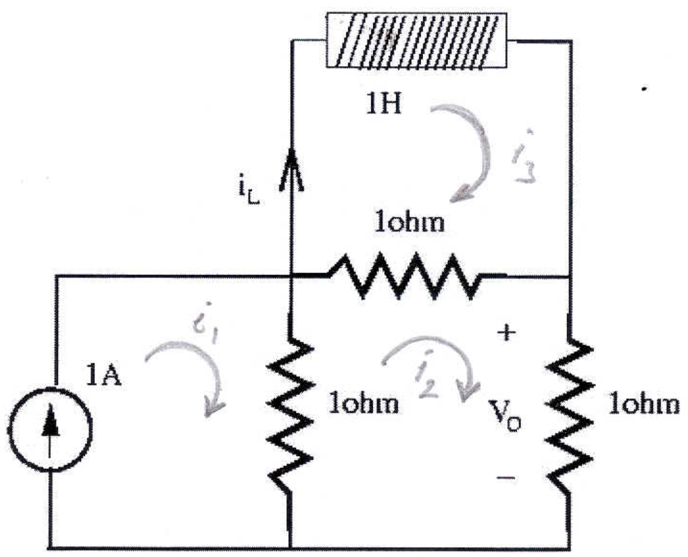
$$300 = 600B_1 - 400B_1 + 5B_2\omega_d = 200B_1 + 300B_2$$

$$-12000 = -36{,}000\, B_1 - 12000\, B_2$$

$B_1 = 0$

$B_2 = 1$

4. (24pnts) For the following circuit, determine $V_O(t)$ for $t>0$, given $\mathbf{i}_L(0)=2A$.



$i_1 = 1A$

$i_2 - i_1 + i_2 - i_3 + i_2 = 0 = 3i_2 - i_1 - i_3 = 3i_2 - 1 - i_3$

$\frac{di_3}{dt} + i_3 - i_2 = 0$

$\Rightarrow \quad 3\left(i_3 + \frac{di_3}{dt}\right) - 1 - i_3 = 0$

$3\frac{di_3}{dt} + 2i_3 = 1$

$\frac{di_3}{dt} = \frac{-2}{3}\left(i_3 - \frac{1}{3}\right)$

$\frac{di_3}{i_3 - \frac{1}{3}} = \frac{-2}{3}dt$

$\ln \frac{i_3(t) - \frac{1}{3}}{i_3(0) - \frac{1}{3}} = \frac{-2}{3}t$

$i_3(t) - \frac{1}{3} = \left(i_3(0) - \frac{1}{3}\right)e^{-\frac{2}{3}t}$

$i_3(t) = \frac{1}{3} + \frac{5}{3}e^{-\frac{2}{3}t} \quad \Rightarrow \quad i_2 = v_o(t) = \frac{1 + i_3(t)}{3} = \frac{1}{3}\left(\frac{4}{3} + \frac{5}{3}e^{-\frac{2}{3}t}\right)$

$= \frac{4}{9} + \frac{5}{9}e^{-\frac{2}{3}t}$

Correct $\tau (= \frac{3}{2}) \to e^{-\frac{2}{3}t}$

10